

J. Bartels, DIS 97
Chicago, April '97

Remarks on

Higher Twist at Small- x_s

Motivation

Theory of higher Twist

Twist-4 in Diffraction

Numerical studies

Motivation

Question to be addressed:

Down to which values in Q^2 (and x_B) can we use DGLAP-evolution?

Pre-HERA point of view:

DGLAP valid for $Q^2 > Q_0^2 \sim 4 \text{ GeV}^2$

Present trend:

Observed rise can be "explained" within DGLAP with flat input at $Q_0^2 < 1 \text{ GeV}^2$

A possible strategy for investigation:

How much space is there for higher twist (twist-4) at small- x and small Q^2 ?

Historical background:

Screening, absorption, ...: concepts of nonperturbative physics, not pQCD
higher-twist: well-defined in pQCD, but more restricted.

Existence of higher twist contributions:

hints from diffractive dissociation near $\beta = 1$

This talk:

status of theory of twist-4
(twist-4 in diffractive dissociation)

first numerical results (qualitative only)

Status of Theory of higher Twist

In the OPE of:

$$\frac{1}{4\pi} \int d^4 z e^{iqz} \langle p | [J(z), \bar{J}(0)] | p \rangle$$

$$= \sum_{i,u} \left(\frac{q^2}{2}\right)^{-u} q_{\mu_1} \dots q_{\mu_u} C_i^u(0) \langle p | O_i^{\mu_1 \dots \mu_u}(0) | p \rangle$$

powers of $(1/q)$ are related to twist $\tau = d - u$:

$$\tau = 2$$

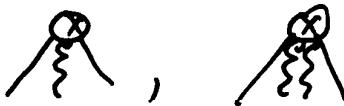


$$\bar{q} \gamma^{\mu_1} D^{\mu_2} \dots D^{\mu_u} q + \text{perm.}$$



$$F^{\alpha \mu_1} D^{\mu_2} \dots D^{\mu_u} F_\alpha^{\mu_1} + \text{perm}$$

$$\tau > 2$$



Ellis, Fornowksi,
Peshier
...

For small-x: gluonic operators should dominate

$$\tau = 4$$



$$\bar{q} \gamma^{\mu_1} D^{\mu_2} \dots D^{\mu_{u+1}} F_\alpha^{\mu_{u+2}} g_{\mu_1 \mu_2} + \text{perm}$$



$$\bar{F}^{\mu_1 \alpha} D .. D \bar{F}^{\mu_2 \alpha} D .. D \bar{F}^{\mu_3 \beta} D .. D \bar{F}^{\mu_4 \beta}$$

$$\bar{F}^{\mu_1 \alpha} D .. D \bar{F}^{\mu_2 \beta} D .. D \bar{F}^{\mu_3 \alpha} D .. D \bar{F}^{\mu_4 \beta}$$

$$\bar{F}^{\mu_1 \alpha} D .. D \bar{F}^{\mu_2 \beta} D .. D \bar{F}^{\mu_3 \beta} D .. D \bar{F}^{\mu_4 \alpha}$$

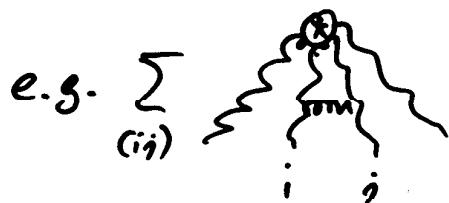
→ 3 different ways of contracting indices

→ complex mixing pattern, number of operators increases with n , triangular structure of anomalous dim. matrix

→ parton correlation

Evolution equations: "quasiparticle operators" Dipatov et al.

- separate evolution equation for each twist (and n)
- to leading order in $\ln Q^2$: only pairwise interactions



- needs nonforward AP splitting kernels

work in progress

JB, C.Boufis,
H.Spiesberger

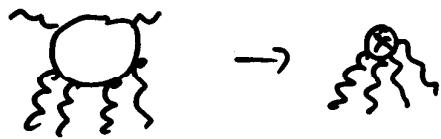
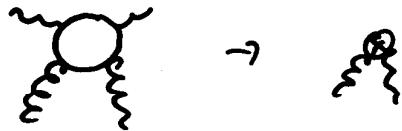
What do we know about twist 4 at scale μ : DLA

"direct" way of calculation

JB, JB+M. Wusthoff

= high-energy / large Q^2 behavior of
scattering amplitudes

Levin, Ryskin, Stavreva



can be analysed to extract anomalous dimensions
and mixing pattern

Some results:

- mixing of 2-gluon twist-four operator and the 3-pow-gluon operators; triangular structure

$$\begin{pmatrix} \theta_2 \\ \theta_4 \end{pmatrix}^R = \begin{pmatrix} 2_{22} & 0 \\ 2_{42} & 2_{44} \end{pmatrix} \begin{pmatrix} \theta_2 \\ \theta_4 \end{pmatrix}^V$$

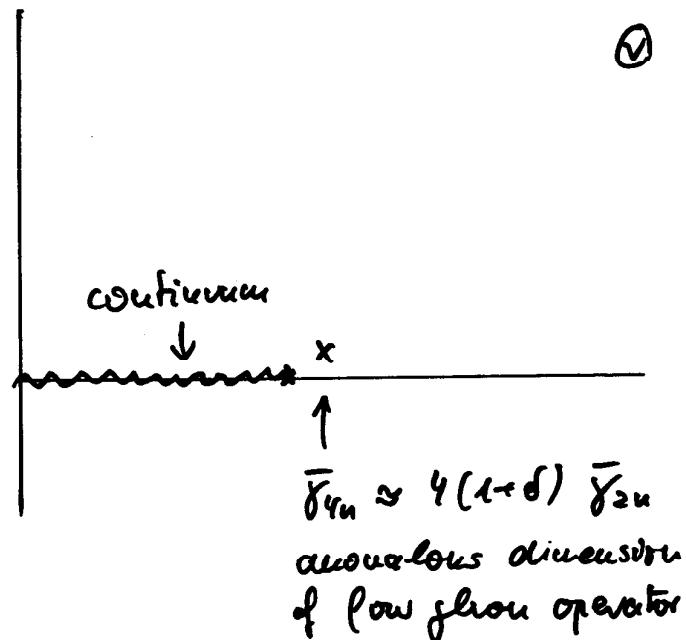
- representation via Mellin transform

$$\text{twist 2: } C_2(Q^2, \alpha_s) \approx e^{\bar{f}_{2u} \ln t/t_0} C_{2u}(Q_0^2, \alpha_s(t))$$
$$t = Q^2/\Lambda^2$$

$$= \int \frac{dv}{2\pi i} e^{v \bar{f}_{2u} t/t_0} \frac{1}{v - \bar{f}_{2u}} C_{2u}(Q_0^2, \alpha_s(v))$$

"anomalous dimension = pole in the v-plane"

twist 4: more complicated structure



explicit formula see below

Lipatov et al.

- alternatively: coupled set of evolution equations can be used to go beyond DLA.

tighter twist in DIS Diffr. Dissociation

tighter twist can be observed directly

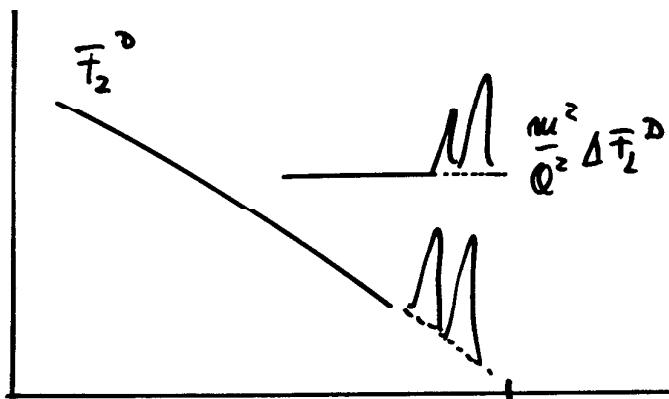
Specific final states can be used for lower bound of $\tau=4$

$$\frac{d^4\tau}{dx dQ^2 dx_p dt} \Big|_{t=0} = \frac{4\pi \lambda_{ew}}{x Q^4}.$$

$$\left\{ \frac{1+(1-y)^2}{2} \left(\bar{F}_t^D + \frac{m^2}{Q^2} \Delta \bar{F}_t^D \right) + (1-y) \left(\bar{F}_L^D + \frac{m^2}{Q^2} \Delta \bar{F}_L^D \right) \right\}$$

↑
dominant
except near $\beta=1$

↑
dominant near
 $\beta=1$



→ few % of total DIS at $Q^2 > 10 \text{ GeV}^2$,
 $x_p \sim 10^{-3}$

First Numerical Studies

JB, C. Bontus

Theoretical input: only gluons, DLA

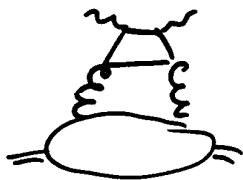
$\tau=2$	two-gluon operator	}
$\tau=4$	few-gluon operator	
	three-gluon operator	
	few-gluon operator	

in double-logarithmic approximation DLA
 $\gamma \sim \frac{\text{const}}{\omega^{-1}}$

Formulae: moment representation ($\omega = u-1$, $t = \ln Q^2/\lambda^2$)

$$\bar{F}_2 = \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^\omega \left[\bar{\Phi}_2(\omega, Q^2) + \frac{Q_0^2}{Q^2} \bar{\Phi}_4(\omega, Q^2) \right]$$

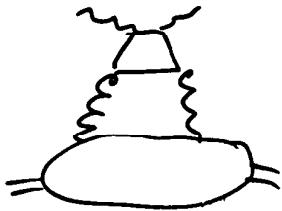
$\tau=2$:



$$\bar{\Phi}_2(\omega, Q^2) = \int \frac{dx}{2\pi i} \left(\frac{x}{t_0}\right)^{\omega} \left[\frac{\alpha_s}{v} + \alpha_s(Q^2) \alpha_s \right] \frac{1}{\omega(v - \gamma_2)} \varphi_{20}(\omega)$$

$$\gamma_2 = \frac{N_c}{\pi \omega} \bar{\alpha}_s, \quad \bar{\alpha}_s = \frac{4\pi}{\beta_0}$$

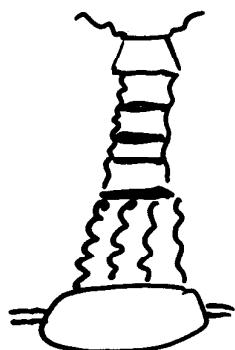
↑
initial
distr.

$\tau = 4$ 

$$\bar{\phi}_4(\omega, \theta^2) = - \int \frac{dv}{2\pi i} \left(\frac{t}{t_0} \right)^v \alpha_s(\theta^2) c_i \frac{1}{\omega(v - \gamma_2)} \phi_{q_0}(\omega)$$

$$\gamma_2 = \frac{N_c}{\pi \omega} \bar{x}_s$$

initial disbr.
flat or
rising



$$- \int \frac{dv}{2\pi i} \frac{1}{t} \left(\frac{t}{t_0} \right)^v \frac{\alpha_s(\theta^2) c_i}{\omega v} \frac{1}{\omega(v - \gamma_2 - 1)}$$

$$. \bar{x}_s^2 \begin{pmatrix} \vec{\alpha} \\ \vec{\beta} \\ \vec{\gamma} \end{pmatrix}^T \left(G^{-1} - \frac{1}{2} S \right)^{-1} \begin{pmatrix} \vec{\alpha}'(\omega) \\ \vec{\beta}'(\omega) \\ \vec{\gamma}'(\omega) \end{pmatrix}$$

\uparrow \uparrow \uparrow
 $4 \rightarrow 2$ gluon
recombination
vertex $q \times q$ - matrix
(3 colors, 3 pairings)
initial disbr.
flat or rising

nontrivial computing problem

minus sign: from AGK-rules

Abrikosov et al

JB, Wüsthoff

JB, Rybníkář

Two different scenarios:

A. Flat (in x) input at Q_0^2 :

- \bar{F}_2 as a place

$$\tau=2 \rightarrow \text{Fig. 1a}$$

$$\tau=4 \rightarrow 1b$$

- sections

$$\tau=4 : x\text{-dependence stronger than } \tau=2 \rightarrow \text{Fig. 1d}$$

$$\sim e^{2\sqrt{c \ln t/t_0} \ln' x}$$

Q^2 dependence deviates from $1/Q^2 \rightarrow \text{Fig. 1e}$

"Semirealistic" situation: $Q_0^2 = 1 \text{ GeV}^2$

$$\bar{F}_2^{\tau=4} / \bar{F}_2^{\tau=2} = 0.1 \text{ at } Q^2 = 10, x = 10^{-3}$$

Very preliminary results:

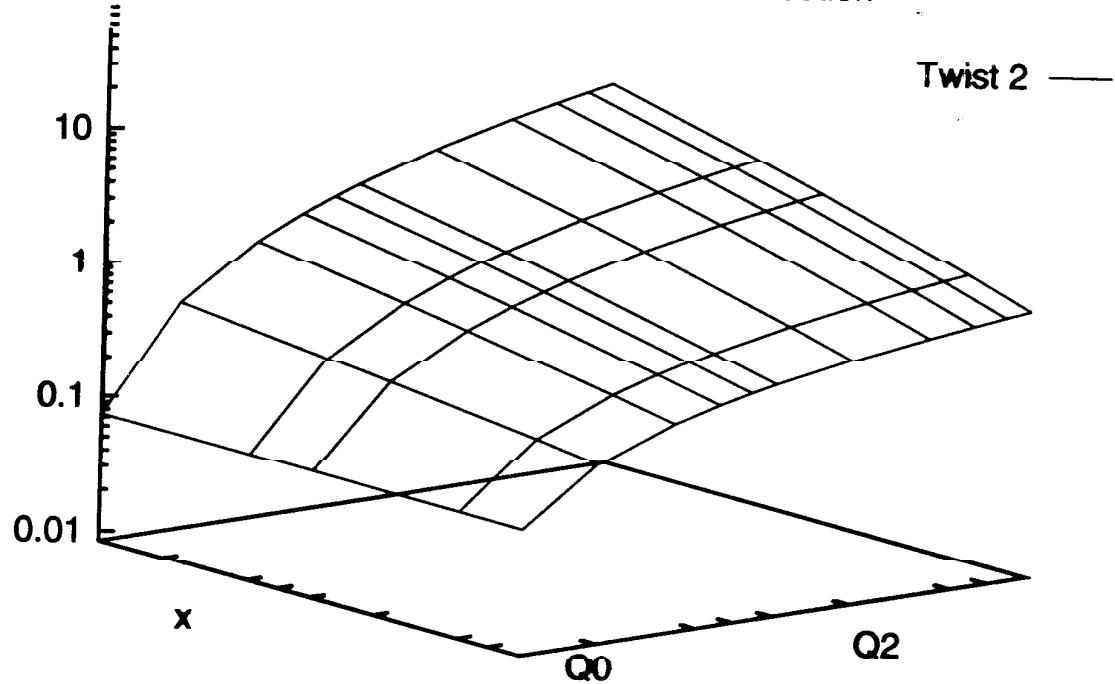
- ratio has doubled at Q^2 $\rightarrow \text{Fig. 2a (d,e)}$
- higher twist becomes smaller than leading twist down to very low Q^2 values

But: how does transition to nonperturbative physics work?

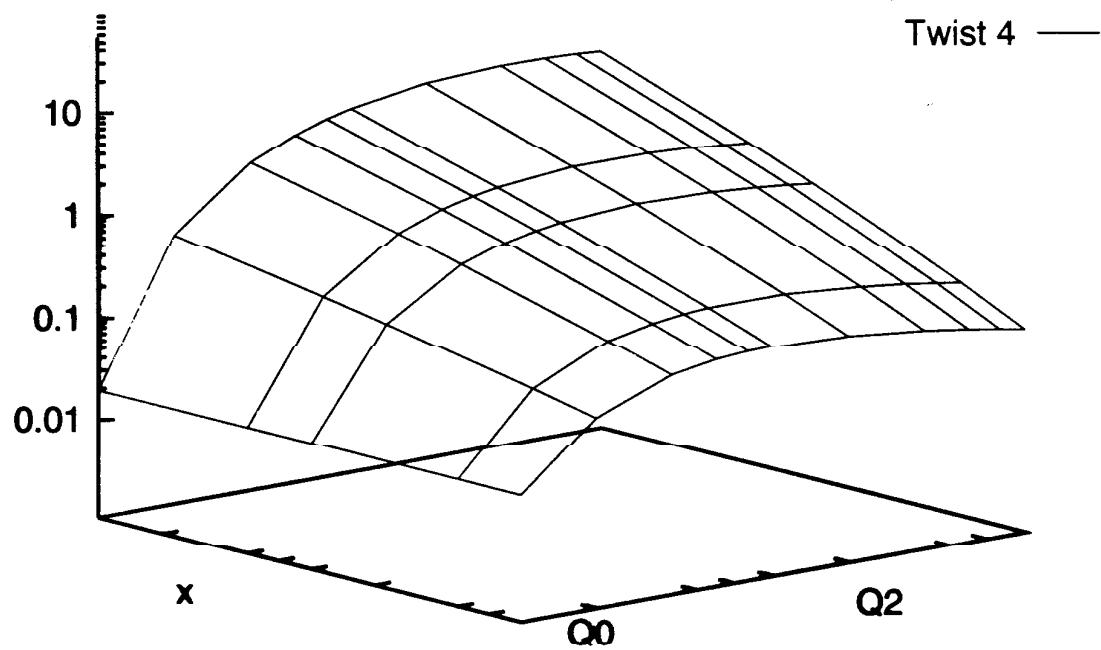
Most plausible: twist-2 \approx twist-4 $\approx \dots$

should get together before key start to fall at small- x .

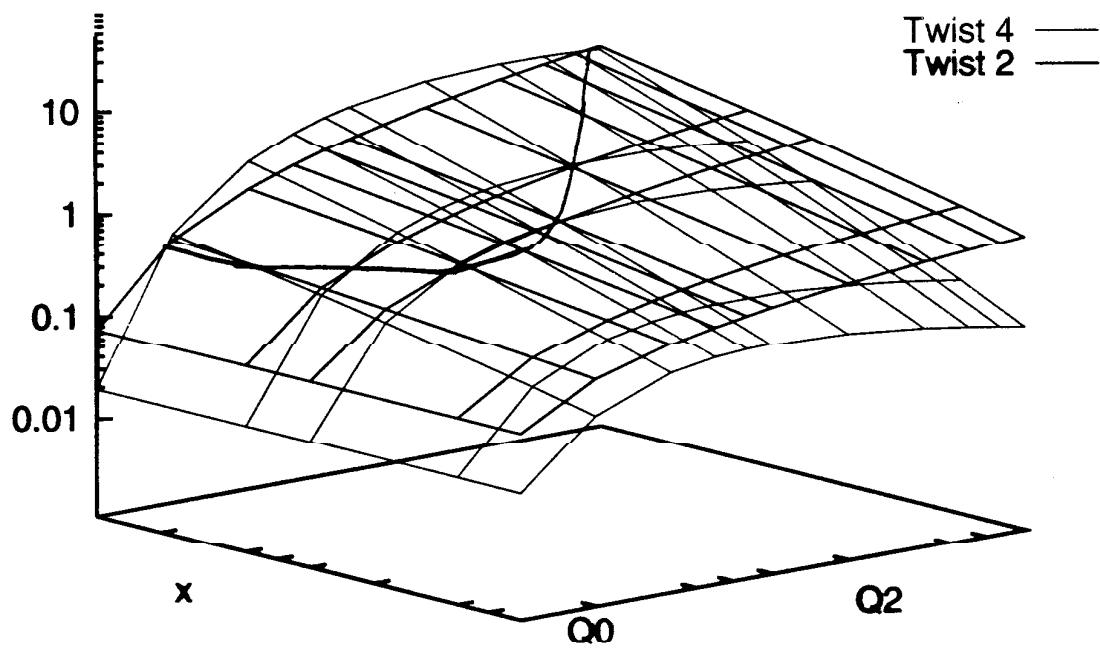
Twist 2 with constant initial distribution



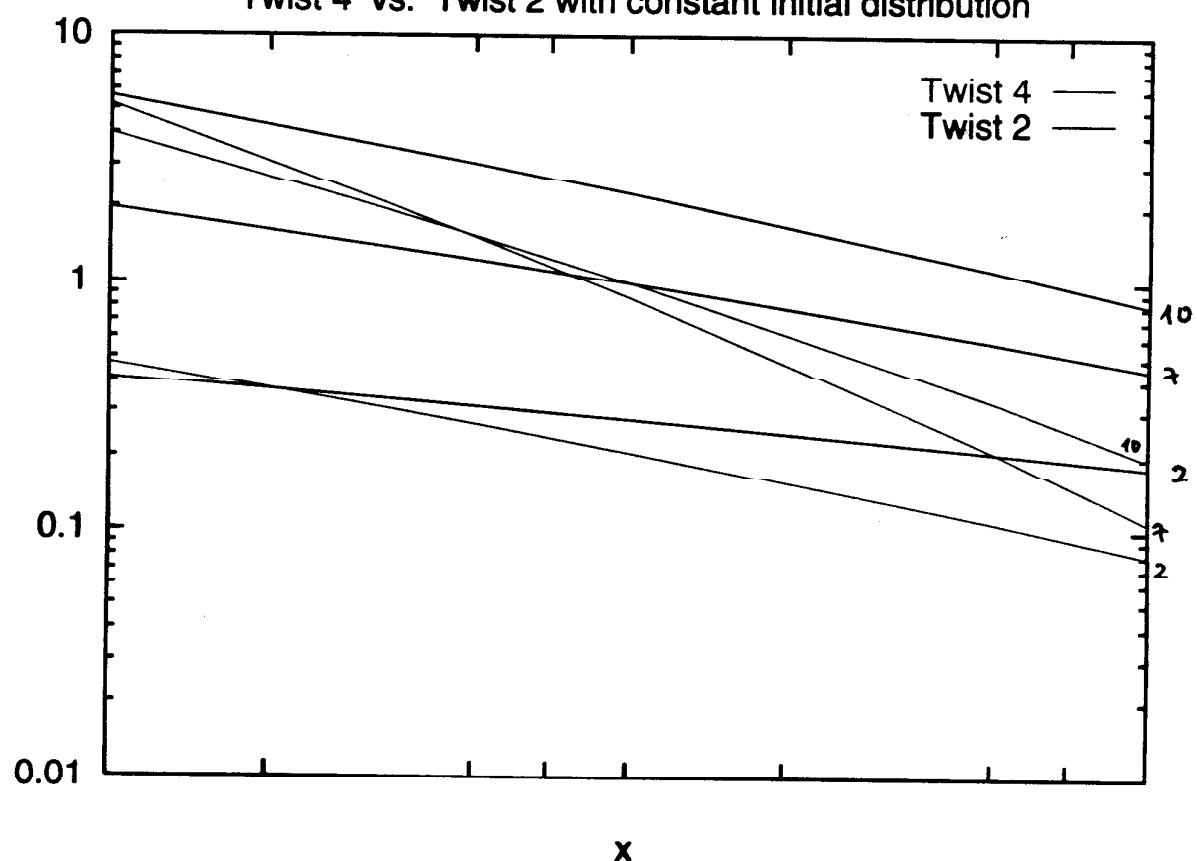
Twist 4 with constant initial distribution



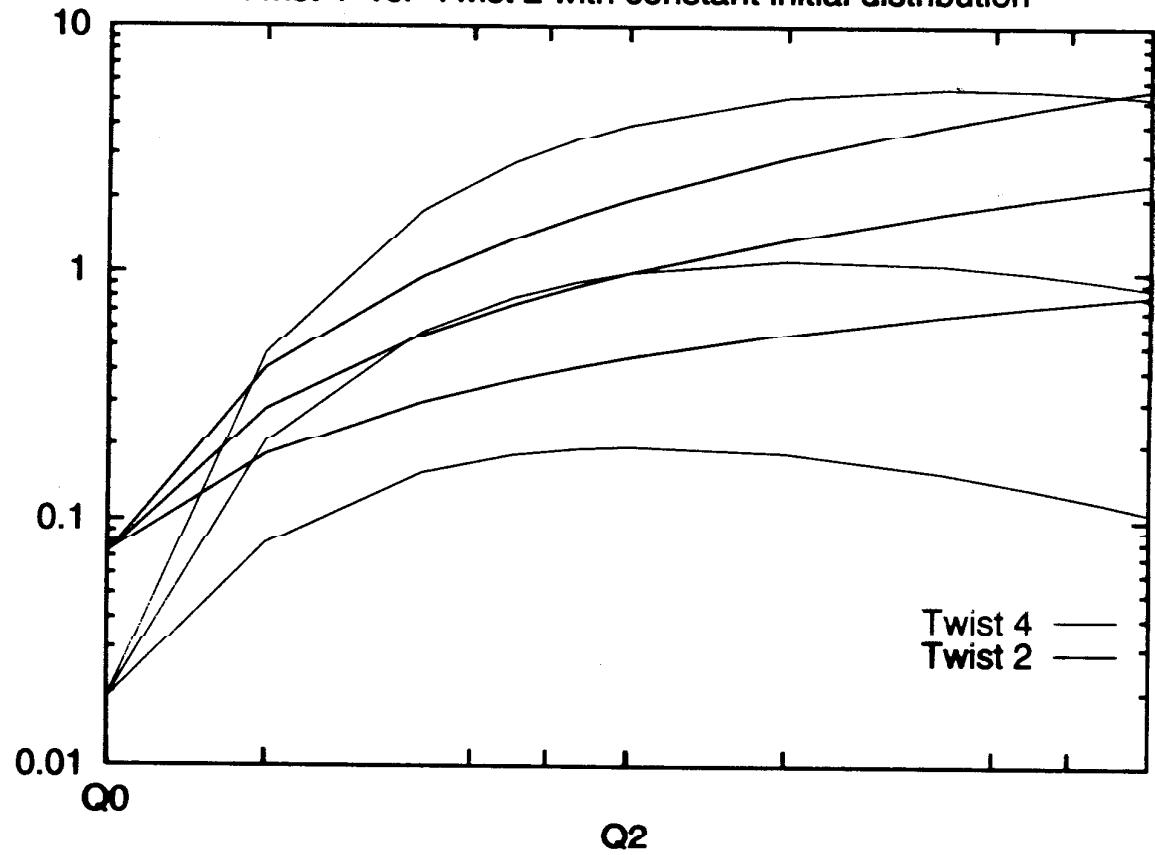
Twist 4 vs. Twist 2 with constant initial distribution



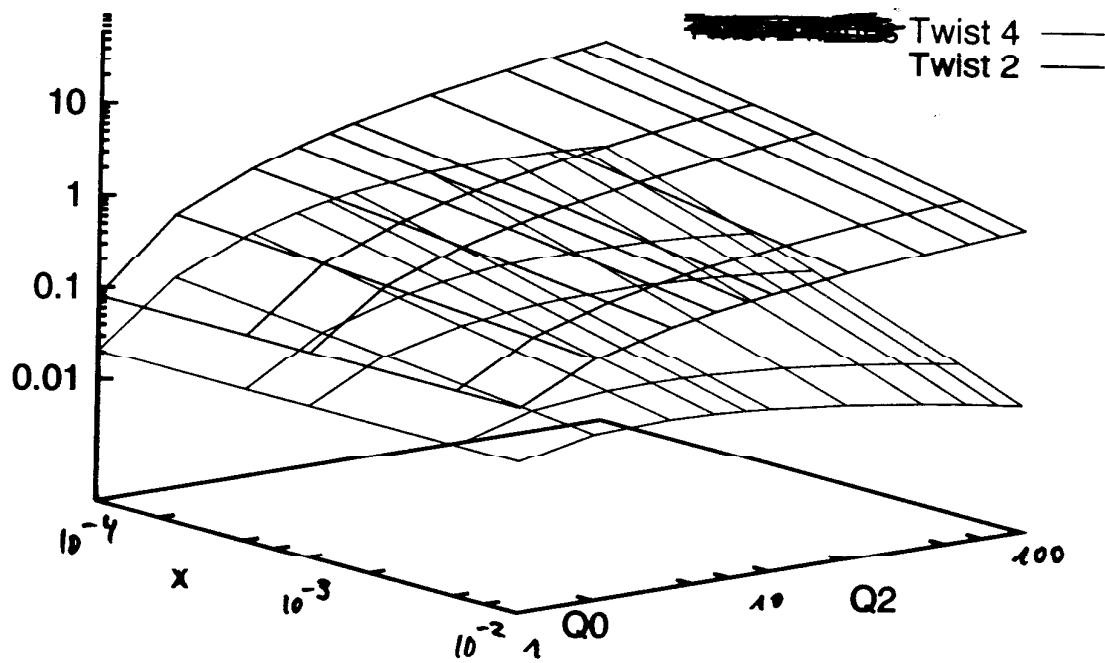
Twist 4 vs. Twist 2 with constant initial distribution



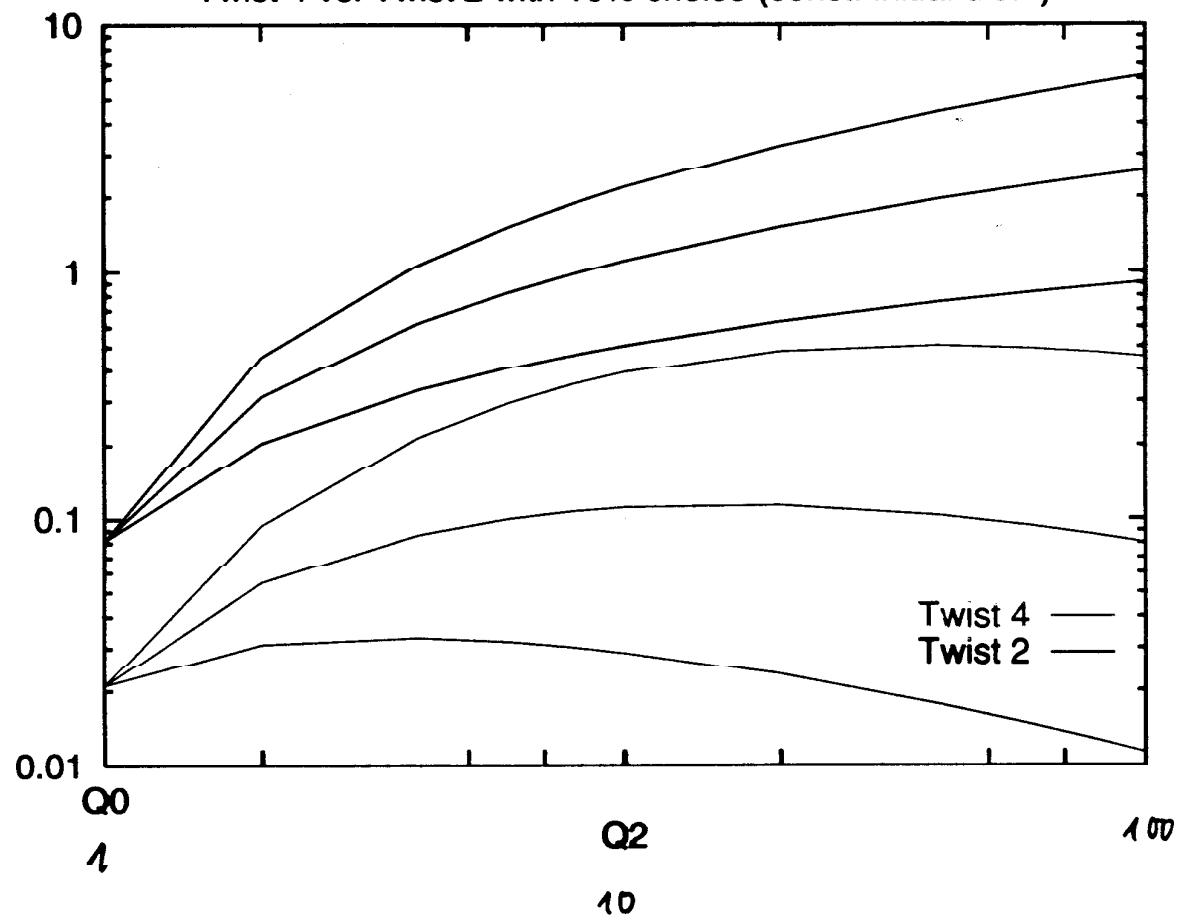
Twist 4 vs. Twist 2 with constant initial distribution



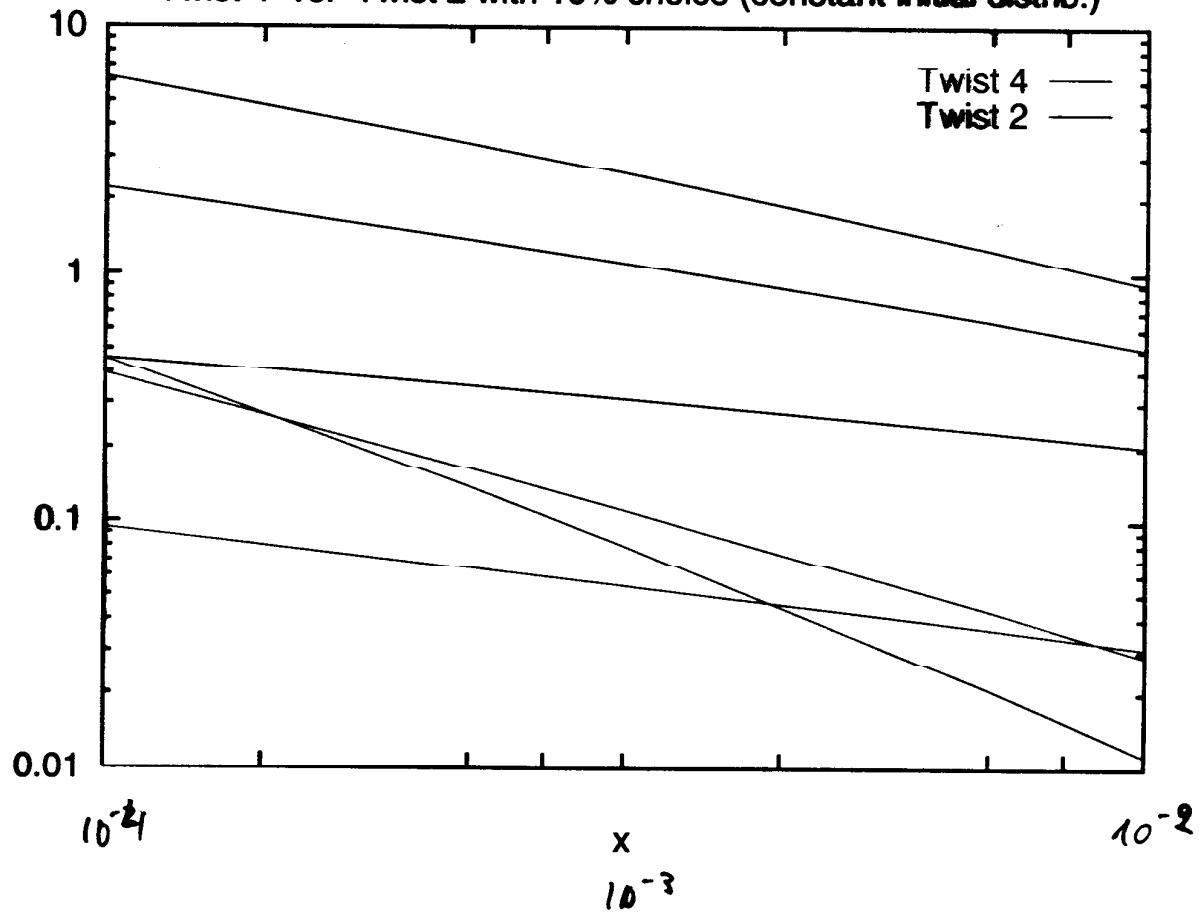
Twist 2 vs. Twist 4 with 10% choice (const. initial distr.)



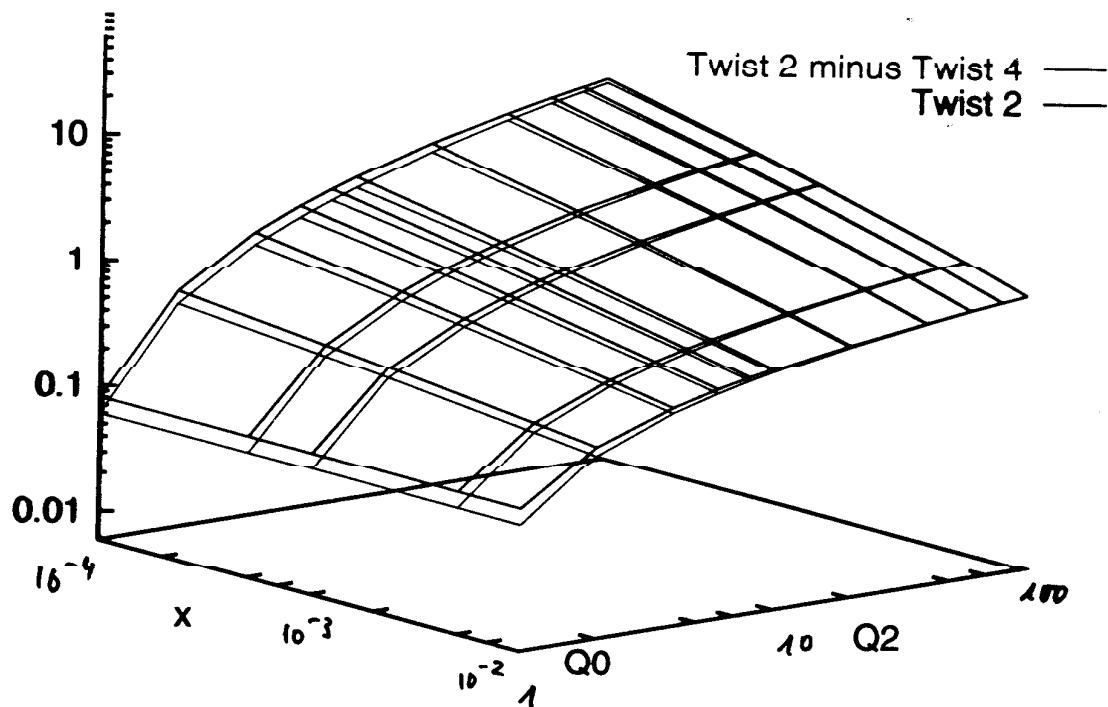
Twist 4 vs. Twist 2 with 10% choice (const. initial distr.)



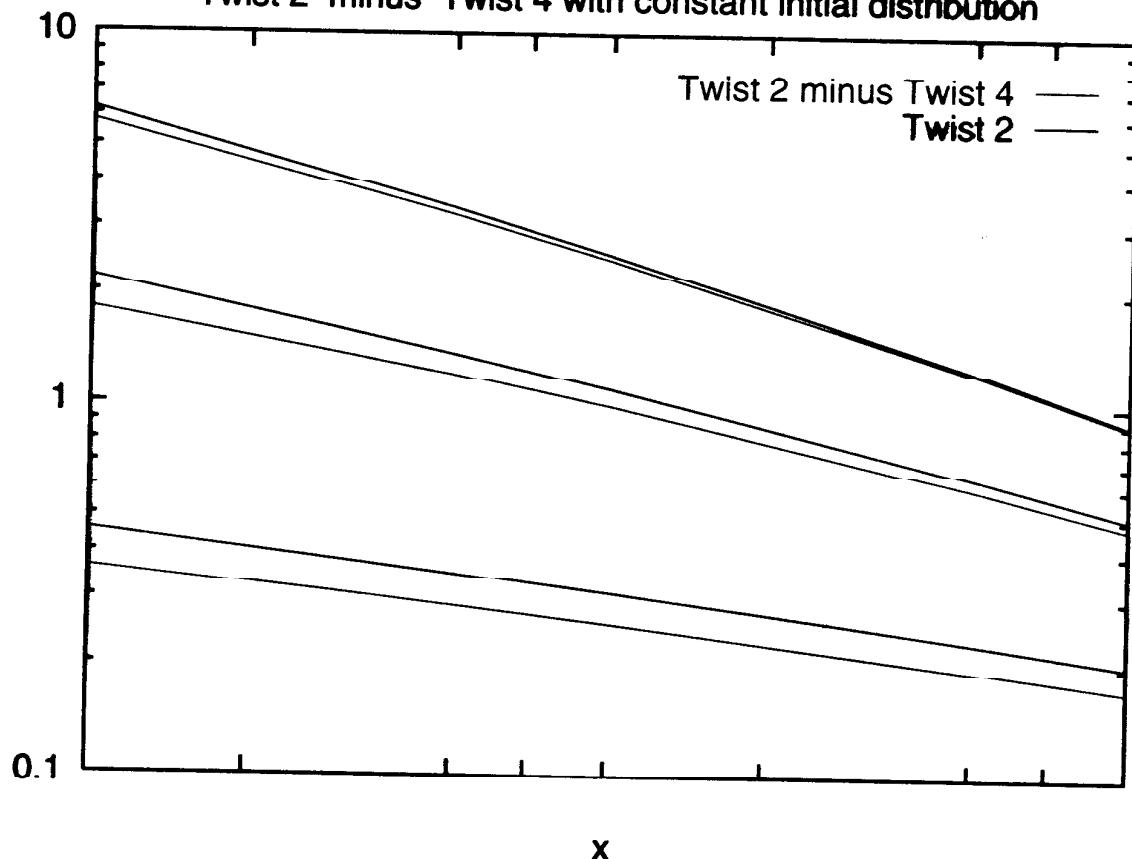
Twist 4 vs. Twist 2 with 10% choice (constant initial distrib.)



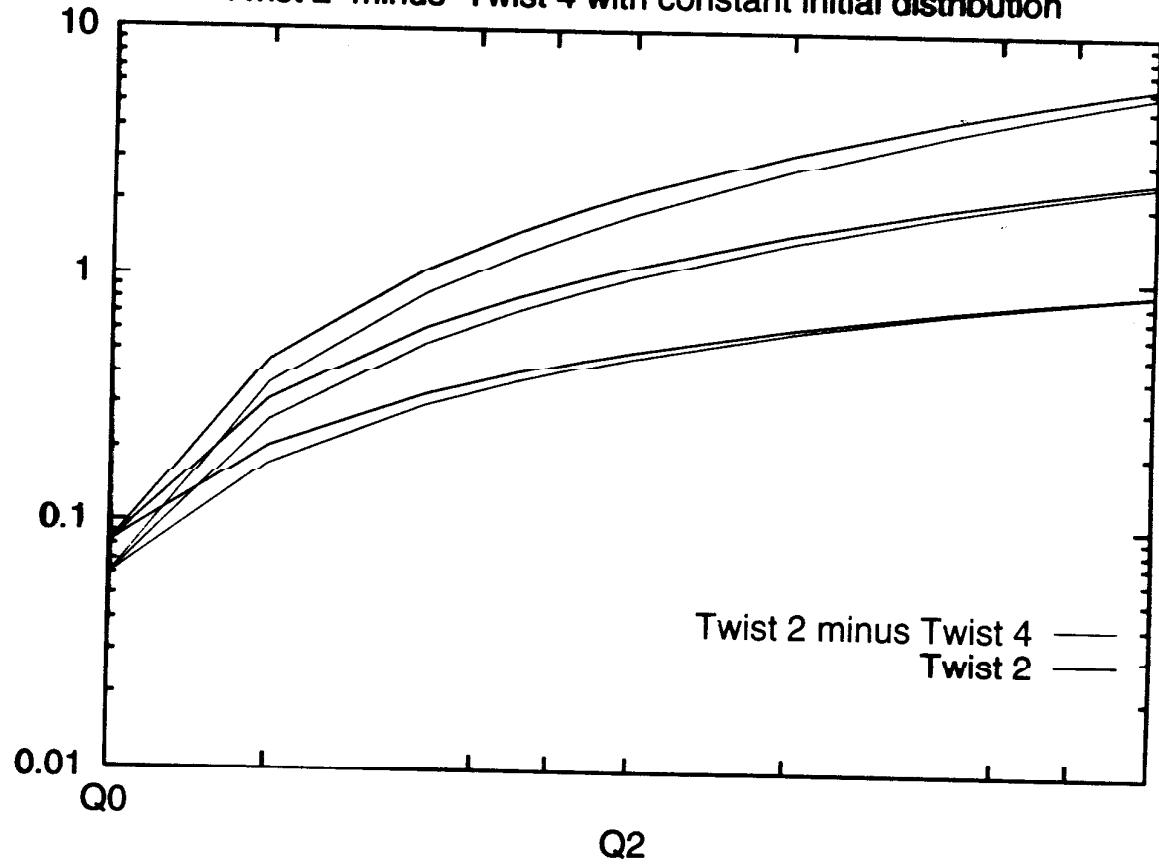
Twist 2 minus Twist 4 with constant initial distribution



Twist 2 minus Twist 4 with constant initial distribution



Twist 2 minus Twist 4 with constant initial distribution



B. Using initial conditions at $Q_0^2 = 1 \text{ GeV}^2$:

$$\text{twist 2: } xg \sim \left(\frac{1}{x}\right)^{0.2}$$

$$\text{twist 4: } \sim \left(\frac{1}{x}\right)^{0.4}$$

$$\bar{F}_2^{T=4} / \bar{F}_2^{T=2} = 0.1 \quad \text{at} \quad Q^2 = 10, \quad x = 10^{-3}$$

Very preliminary results:

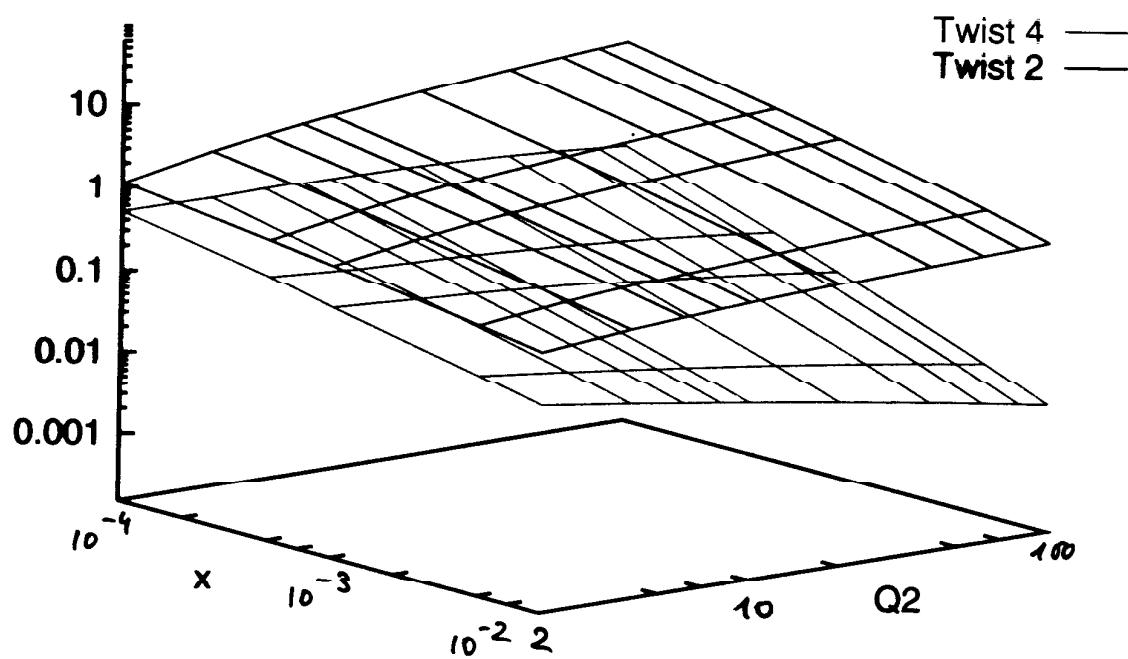
- higher twist keeps growing as Q^2 goes down

$$\bar{F}_2^{T=4} / \bar{F}_2^{T=2} = 0.5 \quad \text{at} \quad Q^2 = Q_0^2 \quad \rightarrow \text{Fig. 3c}$$

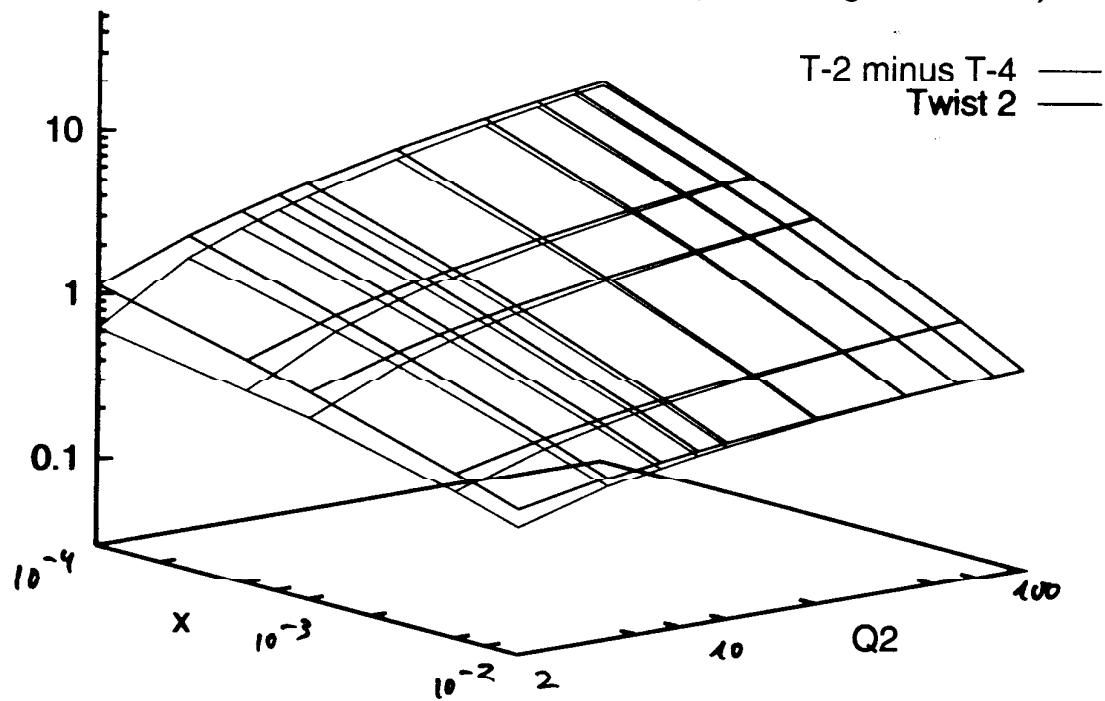
will reach 1 below Q_0^2

- flattens x-rise $\rightarrow \text{Fig. 3b}$
- more realistic for understanding transition to nonperturbative physics

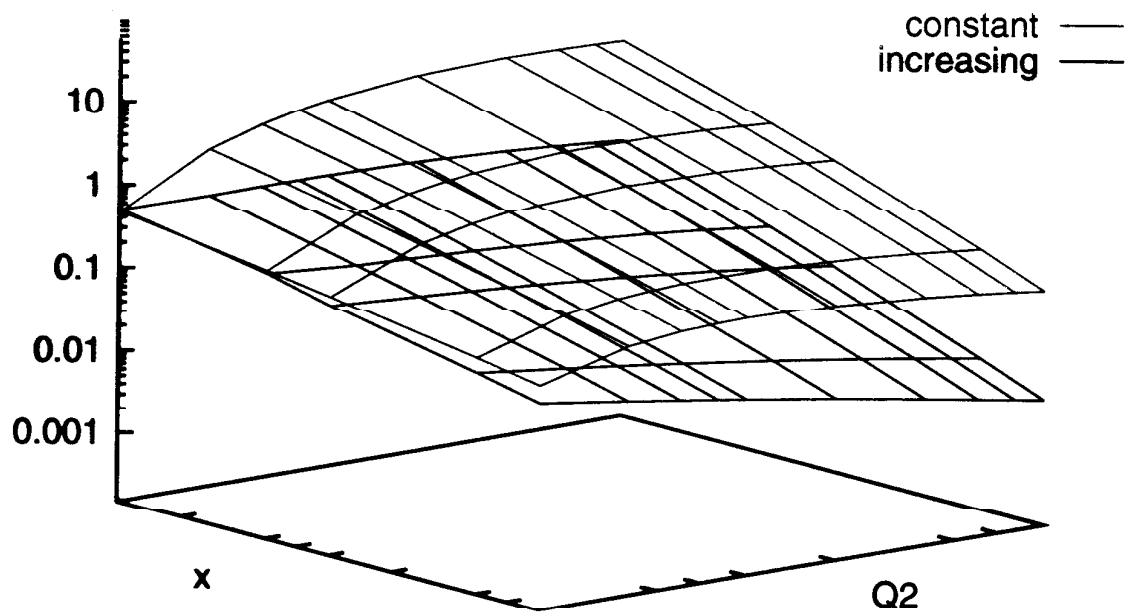
Twist 4 vs. Twist 2 with 10% choice (increasing initial distr.)



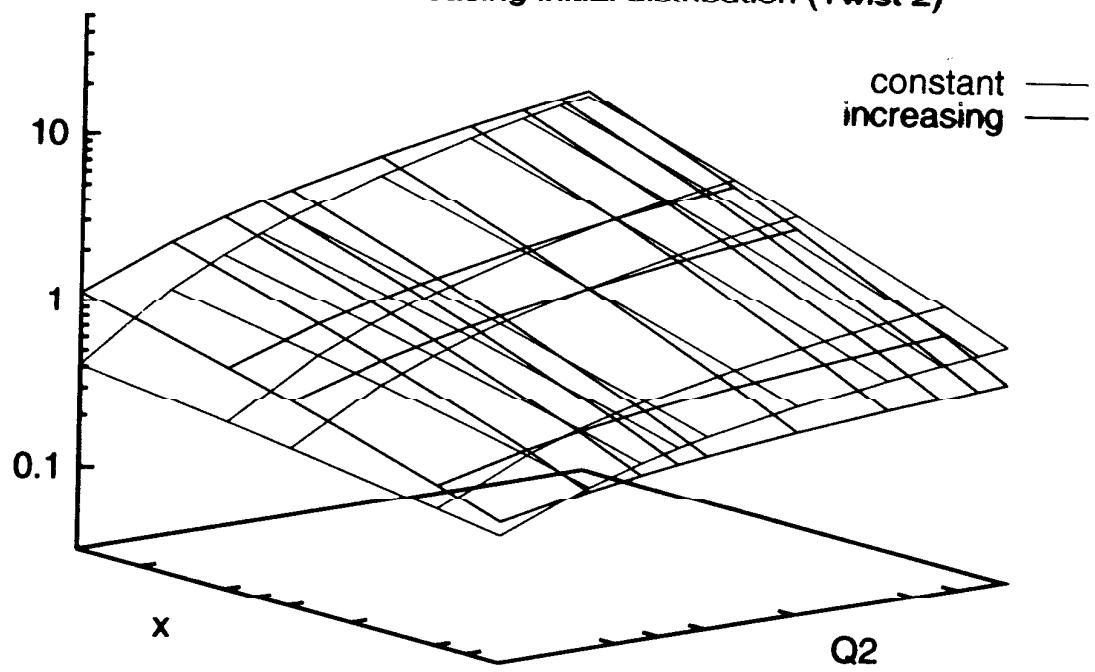
Twist 2 minus Twist 4 with 10% choice (increasing initial distr.)



Constant vs. Increasing initial distribution (Twist 4)



Constant vs. Increasing initial distribution (Twist 2)



Concluding Remarks

- at the beginning of a systematic analysis of twist-4 at small-x
- analytical calculations are needed:
evolution kernels beyond DLA
- computer work is needed
- More scenarios

higher Twist may be not so small
at small x and $Q^2 \sim 1 \text{ GeV}^2$!

Understanding the role of higher twist at
small-x and Q^2 may help to understand
the transition from pQCD to nonperturbative
physics